

Exercise-sheet 1 (October 26, 2015)

1 In-class exercises

1.1 Lattices, lattice vectors and unit-cells

- (a) Which of the lattices in Fig. (1) are Bravais lattices?
- (b) Find a unit-cell with the smallest possible volume and a set of lattice vectors for the linear and two-dimensional lattices given in Fig. (1).

1.2 Symmetries of the square and triangular lattices

- (a) What are the symmetry operations that leave the square and triangular lattices unchanged?
- (b) Consider a square lattice with the following two-atom basis: one atom centered at the origin $(0,0)$, while the other is slightly off site towards the center of the square at $(a/4, a/4)$. Which symmetry operations do you find now?

1.3 Reciprocal lattice and Brillouin zone

- (a) Find the reciprocal vectors of the square lattice.
- (b) Construct the first Brillouin zone.
- (c) What is the set of possible wave-vectors \mathbf{k} lying on the first Brillouin zone, considering periodic boundary conditions?

2 Homework - due date: November 2, 2015 (25 points).

2.1 Sodium in hcp and bcc (5 points)

Sodium transforms from body-centered cubic to hexagonal close-packed (hcp) at about $23K$. Assuming that the density remains fixed through this transition and that the $\frac{c}{a}$ ratio has the ideal value of $\sqrt{\frac{8}{3}}$, calculate the lattice constant a of the hcp phase, given that the cubic lattice spacing is $a = 4.23\text{\AA}$.

2.2 Reciprocal lattice and Brillouin zone of the kagomé lattice (20 points)

- (a) Find the reciprocal vectors of the lattice.
- (b) Construct its first Brillouin zone.
- (c) What is the set of possible wave-vectors \mathbf{k} lying on the first Brillouin zone, considering periodic boundary conditions?
- (d) Draw the allowed wave-vectors within the first Brillouin zone for a 48-site kagomé lattice with periodic boundary conditions.

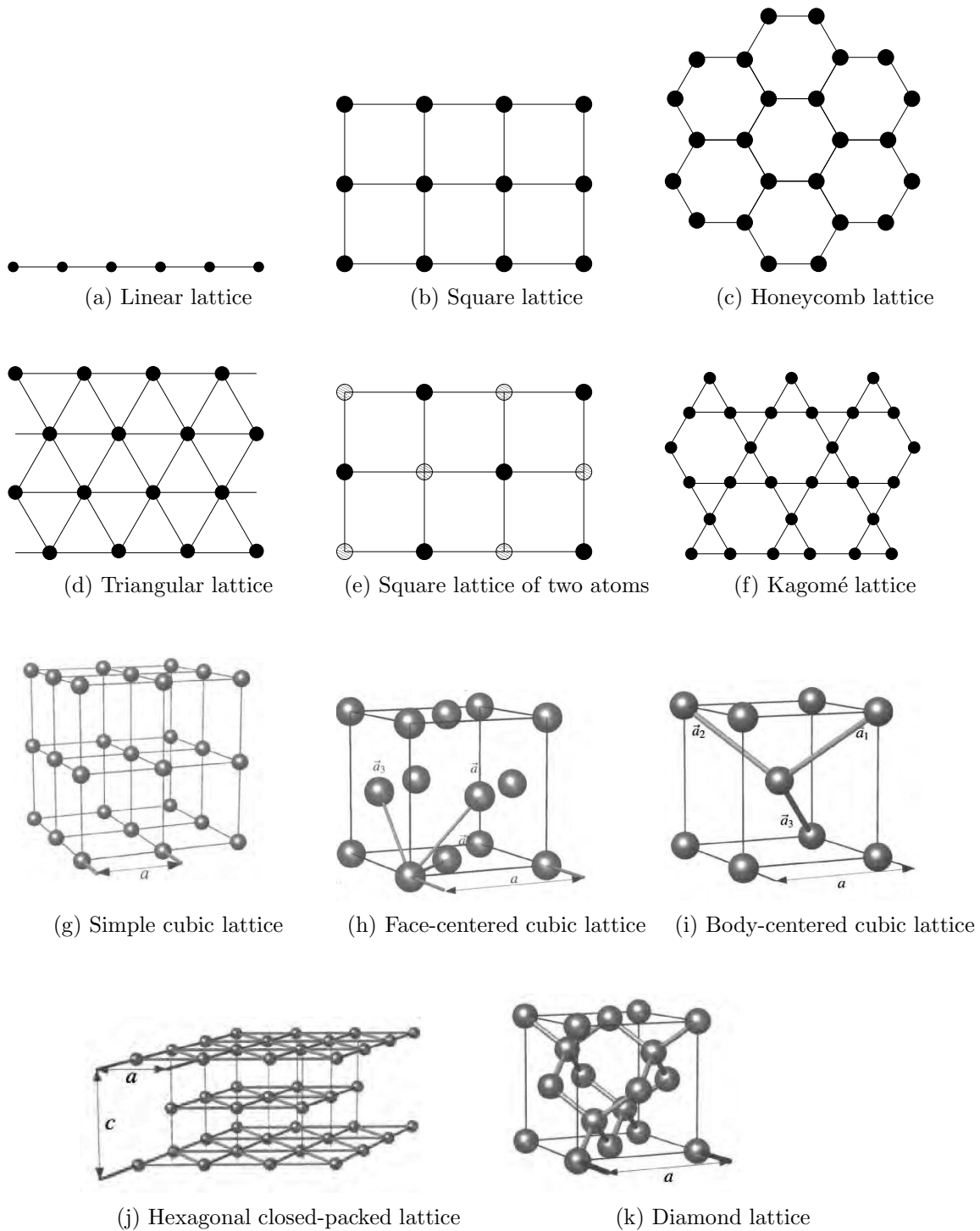


Figure 1: Some lattices. Figures (g)-(k) were taken from *Condensed Matter Physics*, Marder (2010)